Research Topics

1. ANALYSIS AND DIFFERENTIAL EQUATIONS

Problems in differential equations, completely integrable systems, spectral theory and geometric aspects of nonlinear functions. The subjects are strongly related to each other.

Projects:

i. Properties of Solutions of Second-Order Elliptic PDE
Description: We study the existence, uniqueness qnd qualitative properties of solutions of second-order elliptic and parabolic partial differential equations.

ii. Global Geometry of Nonlinear Differential Operators
Description: We study ordinary and partial differential operators as nonlinear functions between function spaces. An example is the operator taking u to u' + u^3 - u, acting on periodic functions, which turns out to be a global cusp, taking points (x,y,v) to points (x, y^3 - xy, v). We make extensive use of singularity theory and infinite dimensional topology.

iii. Topics in Nonlinear Analysis and Spectral Theory
Description: We study spectral properties of matrices, with applications to numerical analysis and integrable systems. In particular, we consider numerical algorithms to compute eigenvalues, parameterizations of special classes of matrices, in a manner compatible to spectral inverse problems.

iv. Regularity in Kinetic Equations
Description: We study the propagation of Lebesgue and Sobolev regularity in kinetic models. The classic example is the Boltzmann equation. The analysis of these models is a mathematical challenge because of its global nature and nonlinear models.

v. Error Prediction for Spectral Methods
Description: An application of the Regularity Theory is the analysis of error prediction, stability and convergence for numerical methods that solve kinetic models such as in spectral methods. These methods are superefficient and adapted to the conservation laws of the problem.

vi. Modeling for Population Analysis
Description: The population models are a modern and well-succeeded application of kinetic equations. We study these models from different perspectives: the existence and uniqueness of the problem, the regularity theory, and other generic properties of the useful model of application.

vii. PDE and Analysis

2. COMBINATORICS

Discrete structures with emphasis on theories of coverage for dimers.

Projects:

i. Combinatorics of Domino Tilings
Description: We study discrete structures mostly related to coverings of quadriculated surfaces by dominoes.

ii. Logic and Combinatorics
Description: Certain results in combinatorics can be stated finitistically (i.e., in Peano arithmetic) but can only be proved using infinite sets.

3. COMPUTER GRAPHICS AND GEOMETRY PROCESSING

Both in academics and industry, computing is a fundamental tool that raises mathematic challenge in the acting and optimization of multidimensional geometric data. These studies require methods from several mathematic fields that include: topology, geometry, combinatorics, functional analysis, and algebra and numeric methods.

Projects:

i. Approximation of invariant curves
Description: Development of estimation methods for curvatures of discrete objects that are invariant under a transformation group action.

ii. Computational Methods in the Characterization of Porous Media in Aquifers.
Description: Aquifers (porous media formed by certain types of rocks containing large amount of water) represent today for 1.7 billion of people around the world the possibility of having access to water in large scale. Besides, under certain conditions, it can be used for CO_2 sequestration. In this multidisciplinary project, applied mathematicians, geologists and geophysicists, from several Brazilian and foreign universities, will use micro and macroscales to characterize several Brazilian aquifers and its possibilities of applications in several areas of social and industrial interest.

iii. Computational topology and mesh structure
Description: Development and applications of topological tools in computational environment. Computerized representation of spaces and topological structures, generally of low dimension, in order to efficiently compute local neighborhoods and topological invariants. Combinatorial tools with topological properties similar to smooth counterparts, with emphasis on discrete Morse theory as formulated by Banchoff and Forman.

iv. Visualization, animation and interface
Description: Mathematical tools for graphical applications adapting performance to the volume of data, guaranteeing the coherence of physical simulation, improving the precision of geometric feature detection, and designing more natural user interfaces and controls.

v. Large Scale Data Visualization
Description: To deal with large amount of data brings today new mathematical and computational challenges. Several areas dealing traditionally with massive quantities of data have as one of its cornerstone to visualize them in a very efficient way. The big challenge is to detect and visualize relevant data information close to real time. Techniques from geometric analysis, computational linear algebra and computer science are at the core of this project. The main application is in social media.

vi. Non-photorealistic rendering

Description: In this project we study novel methods for the computation of hierarchical Poisson disk samplings on polygonal surfaces with applications on Non Photo Realistic rendering (NPR), more specifically, on surface stippling effects.

4. MATHEMATICAL PHYSICS

The Mathematical Physics occupies the space between Theoretical Physics and Pure Mathematics. Mathematically based physical theories, building models with the standard of rigor required of any mathematical area, and creates new mathematical structures.

Projects:

i. Fundamentals of physics
Description: In this Project, we study the mathematical and philosophical aspects of the theories of physics, mainly in quantum mechanics, and general relativity and its classical and quantum form.

5. ALGEBRAIC GEOMETRY

Algebraic Geometry studies properties of spaces locally defined by polynomial equations. Particularly important are properties invariant by birational transformations, that is, invariant by isomorphims over dense open sets, and not necessarily over the whole variety: birational geometry gives rise to nice classifications of curves, surfaces, and higher dimensional varieties. Directly linked with birational geometry is the study of moduli spaces, that is, spaces (variety, schemes, stacks) that parametrize isomorphism (or birational) classes of objects, that can be varieties, vector bundles, sheaves.

Projects:

i. Hilbert schemes of points.
Description: Hilbert scheme of points are one of the simplest and best known examples of moduli spaces of semistable sheaves. Over a smooth algebraic surface, Hilbert schemes of points are smooth and possess an extremely rich geometry, with strong connections with Representation Theory and Mathematical and Theoretical Physics.

ii. Moduli Spaces of Sheaves.
Description: We study semistable coherent sheaves over algebraic varieties and we look for spaces (moduli spaces) that parametrize them in the best possible way. The knowledge of the geometry of moduli spaces of sheaves is very important for the knowledge of the geometry of the underlying algebraic variety and to find example of algebraic varieties with new geometries; that is the case, for example, of compact hyperkähler varieties.

6. DIFFERENTIAL GEOMETRY

Varieties endowed with different structures such as Riemannian metric or foliation, minimal surfaces or constant mean curvature, compact leaves and leaves curvature. The methods used are Geometric, Analytical and Topological.

Projects:

i. Lagrangian Dynamics, global geometry and topology of varieties.

Description: We use tools of Lagrangian and Hamiltonian dynamics: calculus of variations, symplectic geometry and topology, to study the links between conservative dynamics and the topology and global geometry of manifolds. We apply new developments of Aubry-Mather theory, conservative dynamics, rigidity theory in Riemannian and Finsler geometries, foliation theory, geometric group theory and evolution equations for Riemannian and magnetic fields.

ii. Foliations whose leaves have Thurston geometries
Description: Thurston defined the concept of a 'model geometry' in 3-manifolds and showed that there exist exactly 8 such geometries. We study foliations of 4-manifolds such that every leaf has a Thurston geometry.

iii. Affine Geometry
Description: The research project Affine Geometry leads with geometric concepts which are invariant by affine transformations of the n-dimensional space. It includes topics from affine differential geometry and discrete geometry.

iv. Minimal Surfaces and Constant Mean Curvature
Description: Our main goal is to study minimal and constant mean curvature surfaces in three-dimensional homogeneous manifolds, mainly when the ambient space is the product space H2 × R, where H2 is the hyperbolic plane. In fact, we intend to investigate new phenomena and new applications around the theme. More precisely, we are interested in the knowledge and in the development of some geometric aspects of the theory taking into account the following phenomena: the maximum principle, the symmetry and uniqueness, the stability and finite total curvature. We also intend study certain minimal and constant mean curvature type equations under the classical non-parametric analytic PDE viewpoint. Particularly, we specialize to focus on the a priori estimates and on the existence of solutions of the related Dirichlet problems. Finally, we intend to investigate some aspects of the theory of hypersurfaces with some constant symmetric function of curvature. A summary of research results accomplished in 2009-2012 can be found in the site http://www.mat.puc-rio.br/ ~ earp / summary.html.

v. Symplectic Geometry and Group Action
vi. Differential Geometry and Lie Groups

7. PROBABILITY AND STOCHASTIC PROCESSES

The Theory of Stochastic Processes examines the evolution (temporal or special) of systems with random behavior. Its techniques allow extracting the collective behavior of systems composed by a large number of components.

Projects:

i. Stochastic Methods in Finance and Actuarial
Description: We study probabilistic models in finance and non-life actuarial studies.

ii. Nonlinear Fluctuations of Particle Systems
Description: The goal of this project consists in studying the temporal evolution of the fluctuations of the density on weakly asymmetric and totally asymmetric particle systems. The study is focused on the characterization of the dynamical phase transition between universality classes, depending on the strength of the asymmetry.

iii. Phase Transition in partial differential equations
The goal of this project consists in obtaining convergence of weak solutions of partial differential equations with boundary conditions, for which, depending on the parameters given by the boundary conditions, it is

exhibited an interpolation between equations with different qualitative behavior. The approach is based on the underlying particle systems with slow bonds.

iv. Stochastic Modeling in Actuarial Sciences and Financial Markets
Description: The goal of this project consists in performing a theoretical analysis of the properties of distributions that are used in risk theory of actuarial models and the ruin probability and its applications to a data basis of an insurance company with activity in the Portuguese market. In the financial markets we use the theory of martingales to characterize the markets and the price of contracts.

8. DYNAMICAL SYSTEMS

The study of asymptotic behavior of orbits of endomorphism, diffeomorphism and flows, with emphasis on intrinsic properties. The focus is on the stability problems and ways in which this feature disappears.

Projects:

i. Ergodic properties of non-uniformly hyperbolic dynamical systems.
Description: The aim of this project is to determine relations between ergodic properties and those related to hyperbolicity.

ii. Bifurcation and cycles
Description: This project studies the dynamics associated to the unfolding of cycles (homoclinic tangencies, saddle-node-cycles, heterodimensional cycles).

iii. Geodesic flows in manifolds without conjugate points.
Description: We study the following problems:
i. Geometric and topological properties of manifolds without conjugate points whose geodesic flows are expansive.
ii) Mañé's conjecture: Do expansive geodesic flows in compact manifolds have conjugate points?.
iii) Vanishing metric entropy conjecture: If the metric entropy with respect to Liouville measure of the geodesic flow of a compact manifold without conjugate points is zero then the manifold is a flat torus?
iv) Cohomology and subcohomology problems for expansive, non-Anosov geodesic flows, variational ergodic theory.

iv. Lagrangian flows.
Description: We study some problems posed by Birkhoff concerning invariant curves of twist maps in the context of Lagrangian invariant tori in energy levels of Tonelli Hamiltonians. We also study the generic non-existence of Lagrangian invariant graphs in high energy levels of Tonelli Hamiltonians under perturbations by potentials, usually called Mañé´s perturbations.

v. Geometric Theory of Control
Description: We study the application of geometric tools applied to nonlinear control problems. This leads to a coordinate free description of the qualitative features of many nonlinear control models, such as no holonomic mechanical systems. We also use topological methods to understand the theoretical features of certain classes of control systems. Our main tools of investigation are sub-Riemannian geometry, singularity theory, topological tools (e.g. fiber bundle theory, gauge fields etc.) and the theory of nonlinear dynamical systems.

vi. Robust Transitivity and weak hyperbolicity
Description: We study the interaction between transitivity and weak forms of hyperbolicity.

9. TOPOLOGY

The study of topology in foliation theory, group actions and geometry.

Projects:

i. The topology of the space of Locally Convex Curves on the two-sphere
Description: A parametrized curve in the unit sphere of dimension n is locally convex if at every point the derivatives of order 1 to n are linearly independent. A set of curves with prescribed boundary conditions (i.e., position and derivatives up to order n are given at both endpoints) has a rich topology, which depends in a non-trivial way on the initial conditions.

ii. Algebraic sets and Invariant Foliations
Description: We study vector fields (with coefficient in a line bundle) above projective spaces and quotas for the invariant curve hypersurfaces, as well as invariant hypersurfaces under the Pfaff fields.

iii. Asymptotic linking of Rk actions
Description: We study invariants of asymptotic linking of actions of R^k and R^s that preserve the volume in a manifold of dimension k+s+1, or of an action with a foliation. This generalizes work of V. Arnold and B. Khesin.

iv. Stability of Compact Actions
Description: A compact action is a locally free action for which all orbits are compact. We study under what conditions we can guarantee that all perturbations of a compact action are still compact.